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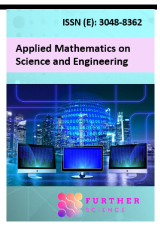

Research Article

A new variant of midpoint Newton's method for solving nonlinear models

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Abstract

Numerous real-world models in engineering and applied sciences frequently involve nonlinear equations that call for trustworthy numerical techniques to solve. Existing iterative methods are constantly being modified, and new ones are being introduced as computational science research expands quickly. Nevertheless, these numerical methods may have high computational cost, but they do have a faster rate of convergence. In this paper, our aim is develop a new variant of midpoint third order iterative methods for solving nonlinear equation. The main theorem proves the third-order convergence, which uses just three function evaluations with $2f$ and $1f'$.

We apply the proposed methods to a number of nonlinear models in the medical sciences, such as the law of blood flow, blood rheology, fluid permeability in biogels, and the human body's temperature control, in order to verify the theoretical results and show its effectiveness. The number of iterations, error in subsequent approximations, computational convergence order (CCO), and CPU time (seconds) are used to assess the method's performance.

1. Introduction

The real world problems in science and engineering including the problems related to computational and applied mathematics, physics, chemistry, fluid mechanics, kinematics, economics, transport theory and medical sciences end up in single valued nonlinear equations of the form $f(x) = 0$ [1]. One of the best root finding method for solving nonlinear equation $f(x) = 0$ is Newton's method. To obtain fast and accurate solutions of nonlinear equations, numerous iterative methods of different order of convergence have been proposed in the last few decades [2–5]. Most of them are the variations or extensions of the famous one-step Newton's method [1] or Steffensen's method [6] in order to improve their convergence order and computational efficiency. Among them optimal iterative schemes that satisfy the conjecture of Kung and Traub [7], perform better than the other root finding methods. However, to develop such methods that achieve an optimal convergence order is not always possible. With the intention to satisfy this conjecture, several authors have proposed optimal root finding techniques, for example, one should see, [8–17].

The classical iterative methods for solving nonlinear equation are, Newton's method (NM), Halley's iteration method, *etc.*, which are respectively given below:

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$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \tag{1}$$

and

$$x_{n+1} = x_n - \frac{2f(x_n)f'(x_n)}{2f'(x_n)^2 - f(x_n)f''(x_n)}. \tag{2}$$

The convergence order of Newton's method is two, and it is optimal with two function evaluations. Halley's iteration method has third order convergence with three function evaluations. Obviously, f'' is difficult to calculate and computationally more costly, and therefore, f'' in Equation (2) is approximated using the finite difference; still, the convergence order and total number function evaluation are maintained [2]. Such a third order method similar to Equation (2) after approximating f'' in Halley's iteration method is given below:

$$y_n = x_n - \beta\frac{f(x_n)}{f'(x_n)}, \quad x_{n+1} = x_n - \frac{2\beta f(x_n)}{(2\beta-1)f'(x_n) + f'(y_n)}, \quad \beta \neq 0. \tag{3}$$

In the past decade, many authors have proposed third order methods with three function evaluations free from f''; for example, let us consider the following third order method, taking $\beta = 1$ in Equation (3):

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}, \quad x_{n+1} = x_n - \frac{2f(x_n)}{f'(x_n) + f'(y_n)}. \tag{4}$$

This is known as Arithmetic mean Newton's method (AM) with cubic convergence (see [3]). This Method (4) is of order three with three evaluations per full iteration having EI = 1.442. The following method is known as Harmonic mean Newton's method (HM) with cubic convergence [18]:

$$x_{n+1} = x_n - \frac{f(x_n)}{2}\left(\frac{1}{f'(x_n)} + \frac{1}{f'(y_n)}\right). \tag{5}$$

The Midpoint Newton's method (MN) with cubic convergence [4] is given by

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}, \quad x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n + y_n)/2}. \tag{6}$$

The Newton-Steffensen method (SM) with cubic convergence [19] is given by

$$x_{n+1} = x_n - \frac{f(x_n)^2}{f'(x_n)[f(x_n) - f(y_n)]}. \tag{7}$$

The efficiency index (EI) of an iterative method is measured using the formula $p^{\frac{1}{d}}$, where p is the local order of convergence and d is the number of function evaluations per full iteration cycle. Kung–Traub [7] conjectured that the order of convergence of any multi-point without memory method with d function evaluations cannot exceed the bound 2^{d-1}, the "optimal order". Thus, the optimal order for three evaluations per iteration would be four. Jarratt's method [20] is an example of an optimal fourth order method. Recently, some optimal and non-optimal multi-point methods have been developed in [5, 14, 21–25] and the references therein.

This paper is organized as follows. Given preliminaries and definitions in section 2. A new third order iterative method is developed by using midpoint rule and also convergence order is analyzed in Section 3. In Section 4, tested application to medical related models to compare the proposed methods with other known methods. Section 5 gives concluding remarks.

2. Preliminaries

Definition 2.1. [26] If the sequence $\{x_n\}$ tends to a limit x^* in such a way that

$$\lim_{n\to\infty} \frac{x_{n+1} - x^*}{(x_n - x^*)^p} = C$$

for $p \geq 1$, then the order of convergence of the sequence is said to be p, and C is known as the asymptotic error constant. If $p = 1$, $p = 2$ or $p = 3$, the convergence is said to be linear, quadratic or cubic, respectively.
Let $e_n = x_n - x^*$, then the relation

$$e_{n+1} = C\, e_n^p + O\left(e_n^{p+1}\right) = O\left(e_n^p\right). \tag{8}$$

is called the error equation. The value of p is called the order of convergence of the method.

Definition 2.2. [27] The Efficiency Index (EI) is given by

$$EI = p^{\frac{1}{d}}, \tag{9}$$

where d is the total number of new function evaluations (the values of f and its derivatives) per iteration.

3. Development of the methods

Let us consider the new variant of third order method with two function and one derivative evaluation,

$$\begin{aligned} y_n &= x_n - \beta \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} &= m_n - \frac{f(m_n)}{f'(x_n)}, \; where \; m_n = \frac{x_n + y_n}{2}, \end{aligned} \tag{10}$$

The proposed method (10) is of order three with three evaluations whereas $2f$ and $1f'$ per full iteration having the efficiency index EI = 1.442, we called as new variant of third order Midpoint Newton's iterative Method ($MNM_{3^{rd}}$).

3.1. Convergence Analysis

Theorem 3.1. *Let $f : D \subset \mathbb{R} \to \mathbb{R}$ be a sufficiently smooth function having continuous derivatives up to third order. If $f(x)$ has a simple root x^* in the open interval D and x_0 is chosen in a sufficiently small neighborhood of x^*, then Method (10) is of local third order convergence*

Proof. Let $e_n = x_n - \alpha$.
Using the Taylor series and we have

$$f(x_n) = f'(\alpha)[e_n + c_2 e_n^2 + c_3 e_n^3 + c_4 e_n^4 + c_5 e_n^5 + c_6 e_n^6 + c_7 e_n^7 + \ldots] \tag{11}$$

and

$$f'(x_n) = f'(\alpha)[1 + 2c_2 e_n + 3c_3 e_n^2 + 4c_4 e_n^3 + 5c_5 e_n^4 + 6c_6 e_n^5 + 7c_7 e_n^6 + \ldots] \tag{12}$$

where $c_q = \frac{f^{(q)}(x^*)}{q! f'(x^*)}$, $q \geq 2$.
Now

$$y_n = \alpha + (1-\beta)e_n + \beta c_2 e_n^2 - 2(\beta(c_2^2 - c_3))e_n^3 + \ldots \tag{13}$$

Let us consider the midpoint between x_n and y_n that is $m_n = \frac{x_n + y_n}{2}$

$$m_n = \left(1 - \frac{\beta}{2}\right)e_n + 1/4\left(4 - 2\beta + \beta^2\right)c_2 e_n^2 + 1/8\left(8c_3 - 4\beta c_3 - \beta^3 c_3 + \beta^2(-4c_2^2 + 6c_3)\right)e_n^3 + \ldots \tag{14}$$

Expanding $f(m_n)$ about α and taking into account (14), we have

$$f(m_n) = f'(\alpha)[(1-\beta/2)e_n + 1/4(4 - 2\beta + \beta^2)c_2 e_n^2 + 1/8(8c_3 - 4\beta c_3 - \beta^3 c_3 + \beta^2(-4c_2^2 + 6c_3))e_n^3 + \ldots] \tag{15}$$

Now, using (11),(12), (14) and (15) in (10) then we have

$$e_{n+1} = \frac{1}{4}\Big((4-\beta^2)c_2\Big)e_n^2 + \Big((-2+\beta^2)c_2^2 + 1/8(16 - 6\beta^2 + \beta^3)c_3\Big)e_n^3 + \ldots \tag{16}$$

Put $\beta = 2$ in equation (16) and simplify, we obtain as follows

$$e_{n+1} = 2c_2^2 e_n^3 + \ldots \tag{17}$$

Hence, the proposed method (10) converges in cubic order. □

4. Numerical examples

This section put forward an extensive quantitative examination of many notable models in the area of medical science, that are expressed as nonlinear equations with one unknown. Due to the inherent nonlinear nature, precise solutions often proved elusive, hence the reliance on iterative techniques is inevitable. The efficacy of the developed method (10) is assessed via performance with contemporary approaches like AM (3), HM (5), MN (6) and SM (7). Numerical evaluations are executed in MAPLE software environment with computations undertaken with 2000-digit precision. The program execution is halted when $|x_{n+1} - x_n| < 10^{-500}$, suggesting convergence. The metric for comparison includes the iteration count $(iter)$ required for convergence, the residual error measured using $(|f(x_{n+1})|)$, (CPU) time in seconds showing computational processing duration, and the computational convergence order expressed as

$$CCO = \frac{ln(|x_{n+1} - x_n| / |x_n - x_{n-1}|)}{ln(|x_n - x_{n-1}| / |x_{n-1} - x_{n-2}|)}, \tag{18}$$

where $x_{n+1}, x_n, x_{n-1}, x_{n-2}$ are four successive iteration results, enabling a good assessment of the method efficacy in addressing non-linearity models that are found in medical field.

4.1. Application to medical related models

Application 1: (Blood rheology model [28])
The blood rheology model is a mathematical expression of the physical and flow nature of blood, which is considered a Non-Newtonian fluid. This model demonstrates that the flow in a tube moves as a plug with minimal deformation, and a velocity gradient occurs near the wall. To investigate the plug flow of Caisson fluid flow, we consider the nonlinear equation:

$$f_1(x) = \frac{1}{441}x^8 - \frac{8}{63}x^5 - 0.05714285714x^4 + \frac{16}{9}x^2 - 3.624489796x + 0.36, \tag{19}$$

where x represents the dimensionless velocity profile of the fluid flow. The implication of the equation above when it is made equal to zero, i.e $f_1(x) = 0$, will connote that the velocity profile has attained a steady-state condition. At this stage, the forces acting on the fluid, such as viscosity and pressure are balanced. The value of x will then suggest the possible steady-state velocity profiles of the fluid flow.

Application 2: (Permeability of fluid in biogels [29])
In a porous medium, the relationship that exist between the pressure gradient and fluid velocity, such as an extracellular fiber matrix, is usually a difficult occurrence that can be explained via nonlinear equation. The equation above acknowledge the specific hydraulic permeability and the radius of the fiber. This gives a good insights into the flow nature or pattern of the fluid. The nonlinear equation that governs this occurrence is expressed as:

$$R_f\left((x^3 - 20p)(1 - x^2)\right) = 0, \tag{20}$$

where R_f denotes the radius of the fiber, that is a valuable parameter used in determining the flow pattern, p denotes the specific hydraulic permeability, which is a measure of the ease with which fluid can flow through the porous medium, x denotes the porosity of the medium, that is a metric of the void space present in fluid flow. Suppose R_f and p are assumed, the equation can then be simplified so as to gain a deeper understanding of the underlying physics. Now, when $R_f = 100 \times 10^{-9}$ and $p = 0.4655$ in the equation, a third-degree polynomial is obtained as:

$$f_3(x) = -100 \times 10^{-9}x^3 + 9.3100x^2 - 18.6200x + 9.3100. \tag{21}$$

The implication when $f_3(x) = 0$, is that the pressure gradient is balanced by the viscous forces. This result to attainment of steady-state flow. Obtaining the value of x (i.e. porosity) in the above equation satisfies the attainment of the steady-state flow condition. In other words, the equation $f_3(x) = 0$ describes the condition under which the fluid flow through the porous medium is balanced.

Application 3: (Population Dynamics and the Law of Population Growth [29])
An important aspect for understanding the behavior of populations in various fields is the study of population dynamics in biology, ecology, and sociology. One basic model describe population growth is the first-order linear ordinary differential equation given as:

$$P'(t) = \lambda P(t) + v, \tag{22}$$

where $P(t)$ denotes the population at any time t, λ is the constant birth rate of the population, v implies the constant immigration rate. The general solution to this differential equation is obtained as:

$$P(t) = P_0 exp(\lambda t) + (v/\lambda)[exp(\lambda t) - 1], \tag{23}$$

where P_0 is the initial population. To determine the birth rate (λ), we would need to have initial condition assign values to the rest parameters. Suppose we adopt same initial condition and parameters values as used in [30], a nonlinear equation is obtained as:

$$f_4(x) = 1564 - 100exp(x) - 435x[exp(x) - 1] = 0, \tag{24}$$

where $x = \lambda$ is the required birth rate. The implication of setting the above equation to zero, i.e $f_4(x) = 0$, is that the birth rate (λ) has reached a critical value that satisfies the given conditions. This critical value of λ represents the rate at which the population grows or declines according to the specified parameters. The solution to the equation $f_4(x) = 0$ provides valuable insights into the population dynamics.

Application 4: (Blood flow law [31])
The Poiseuille's law described the law of blood flow that relates the flow rate of blood via an artery or vein to the viscosity, pressure drop and geometry of the vessel. The nonlinear model that explain this phenomenon is expressed by:

$$f_4(x) = \frac{P\left(R^2 - x^2\right)}{\eta l}, \tag{25}$$

where η is the viscosity of blood, P is the pressure drop, R is the radius of the vessel, x is the distance from the center of the vessel and l is the length of the vessel. The implication of setting the above equation to zero when $P = 4000$, $R = 0.008$, $\eta = 0.027$, and $l = 2$, is that the pressure drop is balanced by the viscous forces. This result to the attainment of a steady-state flow. The solutions to this equation would represent the possible flow rates or velocity profiles of blood flow through the vessel.

Application 5: Body thermal regulation
The formulation of human body thermal regulation was studied by Stolwijk in [32]. Stolwijk theorised the body as a collection of different geometric segments comprising a ball-shaped segment representing the head, five tube-shape segments representing the trunk, hands, arms,

feet, and legs, alongside a blood segment that is a centralised integral to thermoregulatory mechanisms. A crucial rout for heat dispersion is through the respiratory system, emphasizing its usefulness in overall thermal balance. To assessed the thermal energy connected with breath, King and Mody in [33] postulated a mathematical expression correlating the pressure of water vapor in expired breath, P_{H_2O} measured in mm Hg, with the temperature T (^{0}C) of inhaled air, expressed as

$$P_{H_2O} = \exp\left(9.214 - \frac{1049.8}{1.985(32+1.8T)}\right). \tag{26}$$

Given a measured water vapor pressure in expired breath of 0.298 $mmHg$ and letting $x = T$, determining the inhaled gas temperature x necessitates solving the ensuing nonlinear equation:

$$f_5(x) = \exp\left(9.214 - \frac{1049.8}{1.985(32+1.8x)}\right) - 0.298 = 0. \tag{27}$$

Physically, solving for x elucidates the inhaled air temperature corresponding to measured expired breath water vapor pressure, informing models of respiratory heat loss critical in human thermal regulation studies. Numerically, convergence hinges on method robustness and initial estimate proximity to the true root. This mathematical modeling interplay underscores quantitative approaches to human thermophysiology, with respiratory mechanisms pivotal to heat exchange. Accurate determination of parameters like x bolsters understanding of thermal comfort, climate interaction, and clinical thermoregulation contexts. The nonlinear nature necessitates adept numerical strategies for reliable solution approximation, feeding into broader biomedical and environmental health considerations.

The models in Applications 1-5 were solved using the developed method and compared. The computation results, based on metrics already set for comparison of the methods, are detailed in Table (1).

Table 1: Comparison of method's computational results on Applications 1-5

$f_i(x)$	$Methods$	x_0	$iter$	$\lvert f(x_{n+1})\rvert$	CCO	$CPU(s)$	x_0	$iter$	$\lvert f(x_{n+1})\rvert$	CCO	$CPU(s)$
	AM		9	$8.7078e-1039$	3.0029	0.484		6	6.3724e-0922	3.0033	0.318
	HM		8	$1.7181e-1313$	3.0046	0.394		6	3.9652e-1424	3.0042	0.328
$f_1(x)$	MN	2.5	8	$1.6904e-0862$	3.0035	0.437	0.0	6	1.1157e-0924	3.0000	0.342
	SM		8	$1.2219e-0821$	2.9964	0.484		6	9.2764e-0924	3.0000	0.378
	$MNM3_{rd}$		8	$9.1997e-0849$	3.0000	0.493		6	3.5320e-0822	3.0021	0.332
	AM		15	$2.9879e-0868$	2.9524	0.566		16	2.7489e-1114	2.9628	0.541
	HM		12	$2.7464e-0587$	2.9949	0.467		13	1.0802e-0890	2.9966	0.464
$f_2(x)$	MN	2.5	15	$2.9879e-0868$	2.9524	0.606	4.5	16	2.7489e-1114	2.9628	0.556
	SM		15	$2.9879e-0868$	2.9524	0.607		16	2.7488e-1114	2.9628	0.570
	$MNM3_{rd}$		16	$9.8355e-0712$	2.9420	0.620		17	2.1510e-0825	2.9464	0.561
	AM		6	$9.0450e-0528$	3.0345	0.431		8	9.7932e-0572	3.0264	0.496
	HM		6	$9.3366e-0708$	3.0256	0.375		8	1.9726e-1179	3.0153	0.415
$f_3(x)$	MN	2.0	6	$4.1776e-0597$	3.0305	0.419	4.5	8	1.4792e-0810	3.0223	0.421
	SM		6	$2.5225e-0570$	3.0319	0.402		8	1.5003e-0717	3.0253	0.499
	$MNM3_{rd}$		7	$1.0445e-1441$	3.0083	0.438		9	3.9819e-1357	3.0156	0.521
	AM		11	$1.3459e-1366$	3.0022	0.619		13	6.1009e-1054	3.0114	0.655
	HM		8	$7.5317e-0505$	3.456	0.534		10	8.9338e-0693	3.4610	0.497
$f_4(x)$	MN	0.9	11	$1.3459e-1366$	3.0022	0.567	10.5	13	6.1009e-1054	3.0114	0.562
	SM		11	$1.3459e-1366$	3.0022	0.538		13	6.1009e-1054	3.0114	0.605
	$MNM3_{rd}$		11	$2.5531e-0667$	3.0045	0.633		14	1.3662e-1150	3.0026	0.658
	AM		7	$2.0500e-0691$	2.9913	0.368		7	9.4450e-0630	3.0000	0.341
	HM		6	$2.0730e-0632$	2.9953	0.345		7	1.3712e-1055	2.9972	0.342
$f_5(x)$	MN	8.0	7	$9.0750e-0889$	2.9933	0.372	15.0	7	1.0798e-0730	2.9918	0.333
	SM		7	$2.6634e-0810$	3.0000	0.411		7	1.7110e-0693	2.9871	0.348
	$MNM3_{rd}$		9	$3.8894e-0783$	3.0000	0.423		7	2.4078e-0519	2.9825	0.388

4.2. Results discussion

Table 1 presents the computational results of the developed method (10) compared with other methods when used to solve the models $f_i(x), i = 1,2,3,4,5$. The computational results in the residual error column are expressed in the form $A.Be-C$, representing $A.B \times 10^{-C}$, where $A,B,C \in R$. It is observed that the developed method solved all the models just as effectively as the compared methods using the chosen initial guesses. This demonstrates the efficacy of the newly developed method (10) in solving nonlinear models. Although the developed method (10) in some instances, fairly lagged behind the compared methods in terms of precision, required number of iterations ($iter$), computational convergence order (CCO), and CPU time, its novelty cannot be undermined, as its development procedures can pave the way for the creation of several new variants of midpoint Newton's methods. This represents a vital contribution of the developed method (10).

5. Conclusion

A new variant of the midpoint Newton's method with a convergence order of three is developed through the composition of a perturbed Newton's method with an iterative corrector function utilizing the midpoint formula. The developed method was applied to address nonlinear models in the field of medicine, and its performance was compared with existing midpoint methods of the same convergence order. The computational results indicate that the developed method exhibited significant efficacy in solving nonlinear equations. The novelty inherent in the developed method provides fertile ground for the construction of other new variants of midpoint Newton's methods with optimal and higher-order convergence, presenting a promising direction for future research.

Article Information

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